

HAT 3/7/18

WS: Predictable Change

Some sequences, although neither arithmetic nor geometric, still change by a predictable amount. Sometimes, a recursive equation may be obvious. Sometimes, an explicit equation may be obvious. Sometimes, nothing is obvious and we have to explore to reveal the pattern.

1) Finite Differences:

• 1, 4, 9, 16, ...

• 1, 2, 5, 10, ...

• -1, 13, 49, 119, 235, 409, ...

2) Odd/Even:

•  $1, \frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \dots$

•  $\frac{3}{2}, \frac{3}{4}, \frac{1}{2}, \frac{3}{8}, \frac{3}{10}, \dots$

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3) Factorial:

- 1, 2, 6, 24, 120, ...

4) Fibonacci/Lucas Numbers:

- 1, 1, 2, 3, 5, ...
- $t_n = \frac{\sqrt{5}}{5} \left( \frac{1+\sqrt{5}}{2} \right)^n - \frac{\sqrt{5}}{5} \left( \frac{1-\sqrt{5}}{2} \right)^n$
- 2, 1, 3, 4, 7, 11, ...

5) First Order Difference shows a Geometric Pattern:

- 3, 5, 9, 17, ...
- $\begin{cases} t_1 = -9 \\ t_{n+1} = 2t_n + 8 \end{cases}$